Periodic diffraction correlation imaging without a beam-splitter

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Abstract: In this paper, we proposed and demonstrated a new correlation imaging mechanism based on the periodic diffraction effect. In this effect, a periodic intensity pattern is generated at the output surface of a periodic point source array. This novel correlation imaging mechanism can realize super-resolution imaging, Nth-order ghost imaging without a beam-splitter and correlation microscopy.

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References and links

1. Introduction

In recent years, ghost imaging 1995 [1] has attracted wide interest. In traditional thermal ghost imaging experiments, light is separated by a beam-splitter into an object beam illuminated on an object and a reference beam. A bucket detector is used for collecting the output light after the object, and a scanning detector is used for collecting the intensity patterns of the reference beam. The image of the object is hidden by bucket detection, but it can be retrieved by coincidence measurement of the two detectors. The fundamental physics of ghost imaging is the spatially point-to-point correlation of either quantum entangled bi-photons or thermal light. Hence, ghost imaging belongs to correlation imaging. The first experiments of ghost imaging and ghost diffraction with entangled bi-photon pairs generated by spontaneous parameter down-conversion (SPDC) were reported by Shih’s group in 1995 [1, 2]. And the first ghost imaging experiment with classic light source was reported in 2002 [3]. Soon afterwards, ghost imaging experiments with pseudo-thermal light [4–7] and true thermal light [8, 9] were reported. Until now, ghost imaging mechanisms have been investigated in different aspects, including third-order ghost imaging [10–13], two-color ghost imaging [14, 15], ghost imaging by measuring reflected photons [16–18], pinhole ghost imaging [19], ghost imaging through turbulent [20], and ghost imaging under Talbot effect [21, 22], etc.

In all thermal ghost imaging experiments mentioned above, the beam-splitter is an indispensible optical component used for creating point-to-point correlation. While in a quantum ghost imaging scheme, a polarization beam-splitter instead of a beam-splitter is normally needed [1, 2, 15]. Without beam-splitters, the complexity of ghost imaging scheme, especially high-order ghost imaging, can be reduced sharply. In 2008, an interesting ghost imaging mechanism without a beam-splitter named computational ghost imaging [23, 24] was proposed, in which a spatial light modulator (SLM) was applied. Recently, another report on ghost diffraction without a beam-splitter was published [25], in which a pinhole array, instead of a beam-splitter, was placed behind the rotating ground glass to generate thermal light.

In this paper, we propose a new correlation imaging mechanism without a beam-splitter by introducing the periodic diffraction effect. Periodic diffraction effect is an optical effect, in which a periodic intensity pattern is generated at the diffraction plane after a periodic point source array. This effect was observed with both coherent light and incoherent light, resulting from the interactions among the fields from all the point sources. When using incoherent light, the intensity pattern generated at diffraction plane is chaotic but still periodic. Therefore, correlation imaging without a beam-splitter based on periodic diffraction effect is available.
The paper is organized as follows. In section 2, the periodic diffraction effect is studied theoretically and experimentally. In section 3, the new correlation imaging mechanism is proposed, and its features are demonstrated. And in section 4, the conclusion is made.

2. Periodic diffraction effect

In physics, the periodic diffraction effect results from the multiple interference of light fields emitted from a periodic point source array. The periodic point source array can be fabricated through a 2-D grating, such as a pinhole array or a periodically poled LiTaO$_3$ (PPLT) crystal, with input light fields. Each point source is a sub-source. The theoretical derivation of the periodic diffraction effect is similar to that of famous Talbot effect [26–28]. Talbot effect is well-known as the self-imaging of a grating at certain distances. Essentially, Talbot effect focuses on the periodicity of a grating’s diffraction pattern along propagation direction, while the periodic diffraction effect discussed in this paper focuses on the periodicity of the diffraction pattern along directions perpendicular to propagation direction.

According to Huygens wavelets principle, the wave-function $U(z)$ of a point $x$ at the diffraction plane after a 1-D grating illuminated by a coherent light with distance $z$ is

$$U(z) = \sum_{n=-\infty}^{\infty} e^{-i\omega t + ik r_n} r_n^{-1}, \tag{1}$$

where $\omega$ is angular frequency, $k = 2\pi/\lambda$ is the wave number of the coherent light, $r_n = \sqrt{x^2 + (na - x)^2}$ is the distance between the n-th slit and point $x$, and $a$ is the period of the grating. The field amplitude decreases as $r_n^{-1}$ in Eq. (1), resulting from that the light intensity decreases as $r_n^{-2}$ for spherical waves in order to conserve the total probability. In practice, the light intensity $I(z)$ equals to $\langle |U(z)|^2 \rangle$, where $\langle \cdot \rangle$ means statistical average, and has the form as

$$I(z) = \langle | \sum_{n=-\infty}^{\infty} \cos(\frac{2\pi r_n}{\lambda}) r_n + i \sin(\frac{2\pi r_n}{\lambda}) r_n^{-1} \rangle^2. \tag{2}$$

In the traditional Talbot effect, self-images of the grating can be observed at distance $z$ satisfying $\frac{2\pi r_n}{\lambda} = \frac{2\pi}{\lambda} \sqrt{x^2 + (na)^2} = 2p\pi$ for any integer $n$, where $p$ is an arbitrary integer. Note that the wavelength $\lambda$ is normally far smaller than the period $a$ and $z \gg na$ for any integer $n$, so the first-order Taylor expansion of this phase term on $na/z$ is

$$\frac{2\pi}{\lambda} \sqrt{z^2 + (na)^2} \approx \frac{2\pi z}{\lambda} [1 + \frac{1}{2}(\frac{na}{z})^2]. \tag{3}$$

This approximation requires that $z$ has to not overweight $(na)^2/\lambda$, which accounts for that the traditional Talbot effect is a near-field effect observed in Fresnel region. Then, the self-imaging distance or the Talbot distance $z$ is found to satisfy

$$z = \frac{n^2 a^2}{p \lambda}, \tag{4}$$

where $n$ and $p$ are arbitrary integers. Here, the Talbot length $z_T$ is noted as $z_T = a^2/\lambda$. Note that the situation with a 2-D grating is similar.

Whereas, periodic diffraction effect focuses on the periodicity along $x$ axis, not $z$ axis. Under Fresnel approximation, this phase term with a fixed $z$ is

$$\frac{2\pi}{\lambda} \sqrt{z^2 + (na - x)^2} \approx \frac{2\pi z}{\lambda} [1 + \frac{1}{2}(\frac{na - x}{z})^2]. \tag{5}$$
By further Fraunhofer approximation where \( z \gg x^2/\lambda \), the phase term in Eq. (5) is approximated to be \( \frac{2\pi nax}{\lambda z} \), which shows that the pattern’s intensity periodically varies along \( x \) axis satisfying

\[
\frac{2\pi nax}{\lambda z} = 2p\pi,
\]

where \( n \) and \( p \) are arbitrary integers. Then the period \( T \) of intensity pattern along \( x \) axis is found to be

\[
T = \frac{\lambda z}{a},
\]

In order to do some simulation experiments of periodic diffraction effect, the source applied is a \( N \times N \) square point source array with the point-to-point distance \( a \). Here, \( N \) is the point sources’ number in a side of the array. The wave-function \( U(x, y) \) at a point \((x, y)\) in the diffraction plane located apart from the source with a distance \( z \) is

\[
U(x, y) = \sum_{m\in[N]} \sum_{n\in[N]} A(m,n)e^{i\phi(m,n)}\exp[-i\omega t + i\frac{2\pi}{\lambda}r]r,
\]

where \( \{N\} \) is a positive integer set with the maximum integer \( N \), \( A(m,n), \phi(m,n) \) are the amplitude and phase of light emitted from the sub-source \((m,n)\) respectively, and \( r = \sqrt{x^2 + (ma-x)^2 + (na-y)^2} \) is the distance between the sub-source \((m,n)\) and the point \((x,y)\), \( a \) is the period of the point source array. For uniform coherent light source, the lights emitted from all sub-sources have the same amplitudes and phases. While for incoherent light source, the initial amplitudes and phases of any two sub-sources are independent random values and time-varying. Cross-spectral density of the light at the output surface of the generated periodic point source array is

\[
W_k(\rho_{m1,n1}, \rho_{m2,n2}) = \langle A(m_1,n_1)e^{i\phi(m_1,n_1)}A(m_2,n_2)e^{i\phi(m_2,n_2)} \rangle^*,
\]

where subscript \( k = c \) (coherent light) or \( i \) (incoherent light), and \( \rho_{m,n} \) is the position vector of point source \((m,n)\). For uniform coherent light, the cross-spectral density is

\[
W_c(\rho_{m1,n1}, \rho_{m2,n2}) = I_0,
\]

where \( I_0 \) is the average light intensity. While for uniform incoherent light, the cross-spectral density is

\[
W_i(\rho_{m1,n1}, \rho_{m2,n2}) = I_0\delta(\rho_{m1,n1} - \rho_{m2,n2}),
\]

where \( \delta \) is Dirac function. For \( m_1 = m_2 \) and \( n_1 = n_2 \) at the same time \( \delta(\rho_{m1,n1} - \rho_{m2,n2}) = 1 \), otherwise it equals to 0. The generated intensity pattern’s period \( T \) only depends on the effective phase \( 2\pi r/\lambda \). Therefore, no matter with coherent light or incoherent light, the period of the generated intensity pattern is satisfying \( T = \lambda z/a \).

In the simulation experiments, the source array is composed of \( N \times N \) point sources with a central wavelength \( \lambda \), the distance between adjacent point sources is \( a \). A charge-coupled device (CCD) camera used for recording the generated intensity patterns is placed after the source array with a distance \( z \). The CCD contains 200 × 200 pixel cells with each one’s size \( s_{cell} \). Set \( N=20, \lambda=750\text{nm}, a=75\text{um}, s_{cell}=75\text{um} \), then the grey-scale maps detected by the CCD located at \( z=150\text{mm}, 75\text{mm} \) and \( 37.5\text{mm} \) away from the source array with both coherent light and incoherent light are respectively depicted in Fig. 1. The experimental results in Fig. 1 demonstrate that the period of the generated intensity patterns at a same distance with both coherent light and incoherent light are the same. This feature is in good agreement with Eq. (7).
According to the period expression Eq. (7), the period of the generated intensity pattern $T$ only depends on the central wavelength of the source, the detecting distance and the period of the periodic point source array. If the point sources are randomly arranged, the generated intensity pattern will have no periodicity, which is shown in Fig. 2.

3. Thermal-light correlation imaging based on the periodic diffraction effect

3.1. Theory of second-order periodic diffraction thermal-light correlation imaging (PDCI)

In the classical scheme of second-order thermal correlation imaging, pseudo-thermal light generated by a rotating ground glass located after a laser is separated by a beam-splitter into object path and reference path. A bucket detector $D_2$ is used for recording the total transmission or reflected light intensity $I_2$ from the object, which is placed after the beam-splitter with a distance $z_2$ in the signal path. While in the reference path, a scanning detector $D_1$ is placed after...
According to the periodic diffraction effect, the intensity pattern generated by a periodic point source array is periodic. In the other words, the generated intensity pattern possesses the property of point-to-point correlation. Therefore, the periodic diffraction effect can be used for correlation imaging. In the proposed PDCI scheme shown in Fig. 3(a), the beam-splitter is not required. Computational ghost imaging (CGI) [23, 24] does not use a beam-splitter as well, but the cost of PDCI is rather cheaper than that of CGI, because the SLM used in CGI is expensive. An object CCD placed after the object is used as a bucket detector, while a parallel reference CCD is used for recording the reference diffraction intensity patterns. The schematic diagram shown in Fig. 3(b) indicates the proper positions for CCDs in PDCI experiments. In Fig. 3(b), the object CCD is placed at a place marked as a solid line square, and the reference CCD can be placed at any place marked as dotted line squares with shadow. Note that, in our PDCI scheme, more reference CCDs can be introduced in proper positions to realize high-order correlation imaging. The scheme of PDCI is more simple and of flexibility compared with that of conventional ghost imaging, especially for realizing high-order correlation imaging.

The ideal thermal light contents

\[ \langle U_0^*(p_1)U_0(p_2) \rangle = I_0 \delta(p_1 - p_2), \]  

(12)

where \( \rho \) stands for a position vector at the source plane.

According to the diffraction theory under Fraunhofer approximation where \( z \gg \rho^2/\lambda \), the diffraction field-function collected by detector \( D_1 \) is

\[ U_1(p_1) = \int U_0(p_0) \frac{e^{ikz_1}}{i\lambda z_1} \exp[-ik\rho_0 \rho_1 / z_1] d\rho_0, \]  

(13)

and the field-function collected by detector \( D_2 \) in the object path is

\[ U_2 = \int U_0(p_0) \frac{e^{ikz_2}}{i\lambda z_2} \int \exp[-ik\rho_0 \rho_2 / z_2] t(\rho_2) d\rho_2 d\rho_0, \]  

(14)

where \( t(\rho_2) \) is the transmission or reflected function of the object and \( k = 2\pi/\lambda \) is the wavenumber.

The second-order correlation function is known as

\[ G^{(2)}(p_1, p_2) = \langle U_0^*(p_1)U_0^*(p_2)U_2(p_2)U_1(p_1) \rangle. \]  

(15)

Normally, the image can be retrieved by calculating \( \Delta G^{(2)} = \langle \Delta I_1 \Delta I_2 \rangle = \langle I_1 I_2 \rangle - \langle I_1 \rangle \langle I_2 \rangle \), because of the fact that point-to-point correlation lays in the correlation of intensity fluctuation. Substituting Eqs.(13) and (14) into Eq. (15), the function of so-called image is obtained as

\[ \Delta G^{(2)}(\rho) \propto |t(\rho) \odot \text{somb}(\frac{2\pi R}{\lambda z})|^2, \]  

(16)

where \( R \) is the radius of the source, the one-dimension form of \( \text{somb} \) function is \( \text{sinc}(x) = \sin(x)/x \) and \( \odot \) is convolution operator. So the one-dimension function of image is given by

\[ \Delta G^{(2)}(x) \propto |t(x) \odot \text{sinc}(\frac{2\pi R}{\lambda z} x)|^2. \]  

(17)
Next, we consider the resolution of the PDCI. According to Eq. (17), if the object is at point $x = x_1$, then $t(x) = \delta(x - x_1)$ and the retrieved image is an Airy disc described by $\text{sinc}\left[\frac{2\pi R}{\lambda z}(x - x_1)\right]^2$. The radius of the central bright speck is $\frac{\lambda z}{2R}$, which is the minimum distinguishable length or resolution of the optical setup of the conventional ghost imaging according to Rayleigh’s criterion. While in the PDCI experiments, the source is a $N \times N$ mutual-independent point source array with a period $a$. By theoretically analysis, the resolution of PDCI is

$$\text{Res} = \frac{T}{N} = \frac{\lambda z}{aN},$$

where $aN$ is the side length of the source array.

Consider that the object is a point corresponding to one pixel cell of object CCD, both the CCDs are composed of 200 $\times$ 200 pixel cells with each one’s size $s_{cell} = 75 \mu m$, and the period of the point source array $a=75 \mu m$. The mutual-independent point source array is achieved by resetting the amplitude and phase of each point source with statistically independent random numbers for each record. The imaging distance $z=75mm$, and the corresponding period of the generated intensity pattern $T=7.5mm$. The intensity patterns recorded by the object CCD and the reference CCD are denoted by $I_o$ and $I_r$, respectively. The image of a point, corresponding
to the object CCD’s pixel cell (50,50), is retrieved by calculating $\langle \Delta I_o(50,50) \Delta I_r \rangle$ with 2000 records. The retrieved images are shown in Fig. 4. The image’s size depends on CCDs’ sensing area, which is 15mm × 15mm. Therefore, the retrieved image of the point (50,50) is composed of four bright specks, as shown in Fig. 4. Here, Fig. 4(a) is the retrieved image with a 10 × 10 point source array, while Fig. 4(b) is the retrieved image with a 20 × 20 point source array. The resolutions of PDCI in Fig. 4(a) and Fig. 4(b) are 0.75mm and 0.375mm, respectively, which are in good agreement with resolution expression Eq. (18).

![Fig. 4](image)

Fig. 4. (a) The retrieved image of a point with a 10 × 10 point source array. (b) The retrieved image of a point with a 20 × 20 point source array.

Similarly, the PDCI experiments with a more complex object are achieved through simulations. We choose the object as a mask with a Chinese character meaning wood curved on its left top corner, shown in Fig. 5.(a). The size of the mask is the same to that of a CCD’s sensing area, i.e. 15mm × 15mm. The light source is a 20 × 20 point source array with $a=75\mu m$, and each CCD is a 200 × 200 pixel cell array with $s_{cell}=75\mu m$. The bucket detector is achieved by recording the sum of all intensity elements of matrix $I_o$ for each time. The images of the mask can be retrieved by calculating $\langle \Delta (\sum I_o(m,n)) \Delta I_r \rangle$, where $m,n=1,...,200$. Then, the retrieved images of the mask with distance $z=75mm$ and 150mm are shown in Fig. 5.(b) and Fig. 5.(c), respectively. Similar to the point-imaging experiments, four characters are obtained in Fig. 5.(b) and only one character is obtained in Fig. 5.(c). Compared with the image in Fig. 5.(c), the image in Fig. 5.(b) is more clear because it has a higher resolution according to Eq. (18).

![Fig. 5](image)

Fig. 5. The mask and its images retrieved by PDCI with 2000 records at distance $z=75mm$ and 150mm, respectively.

The resolution of PDCI can be easily improved according to Eq. (18). If in the experiment, the source is a 1000 × 1000 point source array with $a=75\mu m$, the imaging distance $z=75mm$. Then the resolution will be 7.5um, and it can be further improved by decreasing the imaging distance $z$. Therefore, the PDCI is capacity of realizing super-resolution imaging in theory. However, the imaging resolution of PDCI is restricted by the finite size of CCD. Because given
a CCD with a fixed size, more duplicated images will be collected by using a shorter distance and each of them will be detected by the smaller amount of CCD pixels.

What’s more, if the generated intensity pattern’s period $T$ is smaller than half of the size of CCD’s detecting area, then the PDCI can be realized with only one CCD. Part of the CCD array is used as a bucket detector, and the other part is used as a scanning detector. In addition, the PDCI has a higher resolution at a closer imaging distance in theory. Therefore, a new imaging technology similar to microscopy based on spatial correlation, named correlation microscopy, can be realized by PDCI, although the resolution reported in this paper is still far away from the useful microscopy applications. This work is being investigated.

3.2. High-order PDCI

One distinguished advantage of our scheme here is that high-order correlation imaging is easily realized by PDCI. According to the investigation on arbitrary-order lensless ghost imaging, the image visibility can be enhanced as the order $N$ increases [12]. The high-order PDCI experimental setups are simple and similar to Fig.3. Take the third-order PDCI for example, in the experiments, three CCDs are used: an object CCD ($D_1$) and two reference CCDs ($D_2$ and $D_3$). $D_1$ is placed after the mask, while $D_2$ and $D_3$ are placed in any two positions marked by dotted line square in Fig.3.(b). The intensity patterns synchronously recorded by $D_1$, $D_2$ and $D_3$ are $I_1$, $I_2$ and $I_3$, respectively. Then the third-order correlation function of the image is given by

$$G^{(3)} = \langle \int t(\rho)I_1(\rho)d\rho I_2I_3 \rangle,$$

(19)

where $t(\rho)$ is the transmission function of the mask. The simulation of the third-order PDCI is similar to the second-order PDCI, except that a third intensity matrix $I_3$ is introduced in the third-order PDCI. And it’s straightforward to extend to the Nth-order PDCI.

The third-order and the fourth-order PDCI experiments were simulated. The parameters in these experiments are the same to that in the second-order PDCI. The images retrieved by the second-order PDCI, third-order PDCI and fourth-order PDCI with 2000 records are shown as the figures in the first row of Fig.6. Clearly, the quality of the image obtained is improved with the increasing of the order $N$, which is in good agreement with the theory [12].

At the same time, an interesting phenomenon was found that the quality of the image retrieved by calculating $\Delta G^{(N)}$ is not improved with the increasing of the order $N$. The fluctuation-considered Nth-order correlation function of the image is given by

$$\Delta G^{(N)} = \langle \Delta \int t(\rho)I_1(\rho)d\rho \Delta I_2 ... \Delta I_N \rangle.$$

(20)

Similarly, the second-order, third-order and fourth-order images retrieved by calculating Eq. (20) are shown as the figures in the second row of Fig.6. Clearly, the ranking from the best to the worst on image visibility is: the second-order PDCI, the fourth-order PDCI and the third-order PDCI. This result is different to the theory in the article [12], because in which the image is retrieved by calculating $G^{(N)}$. Besides, the ranking hints that the quality of odd-order correlation imaging is worse than that of contiguous even-order correlation imaging, because the intensity fluctuations recorded by CCD pixel cells are random numbers of zero-mean. They might be positive or negative in a record. Then in each record, the products of intensity fluctuations of mutual-correlated pixels are always positive in the even-order PDCI, but not sure in the odd-order PDCI. Therefore, the visibility of image retrieved in the even-order case is better. Besides, it’s noted that the image retrieved by calculating $\Delta G^{(2)}$ has the best quality in our experiments.
3.3. Shape merging effect in the PDCI

Next, we demonstrate the shape merging effect in the PDCI. To explain this effect better, we make a definition of the non-repeated diffraction pattern (NDP). An arbitrary square area with its side length equal to the period $T$ in the periodic intensity pattern is named a NDP.

According to the periodic diffraction effect, the image of the character in the PDCI reappears in the imaging plane with a period of $T$. That’s why only one character image is obtained in Fig. 5.(c), and four character images are obtained in Fig. 5.(b). If the size of the object is larger than that of a NDP, then a shape-merged image is obtained in each NDP. In the following simulation experiment, a mask with English letters ‘sj’ at top left corner and with English letters ‘tu’ at lower right corner, shown as the Fig. 7.(a), is placed with distance $z = 75\text{mm}$. The other experimental parameters are the same to that in previous second-order PDCI experiments. Because of the relative position relation between ‘sj’ and ‘tu’, a merged ‘sjtu’ is retrieved by the PDCI, shown in Fig. 7.(b). This effect of the PDCI is named shape merging effect.

Fig. 6. First row: the images of the mask retrieved by calculating $G^{(N)}$, where the order $N=2,3$ and 4 respectively. Second row: the images of the mask retrieved by calculating $\Delta G^{(N)}$, where $N=2,3$ and 4 respectively.

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Fig. 7. The mask and its image retrieved by the PDCI.
The shape merging effect extends the PDCI to more potential applications, such as image reconstruction or image encryption. For example, in the scheme of the image encryption based on the PDCI, two bucket detectors used as senders and a scanning detector used as the receiver are placed with a separation of $T$ in the imaging plane. The complete image can only be obtained by coincidence measurement between the sum of total intensity values detected by the senders and the intensity distribution detected by the receiver. The intensity informations detected by the two senders are actually the secret key to each other.

The PDCI provides an effective physical method to realize the image reconstruction and the image encryption, which are normally realized by software methods.

4. Conclusions

A new imaging method named periodic diffraction correlation imaging (PDCI) is proposed for the first time. The PDCI is based on periodic diffraction effect, in which the period of generated intensity pattern is $T = \lambda z/a$. The PDCI can realize the ghost imaging without a beam-splitter with a cheap cost. The PDCI can further realize both the super-resolution imaging and the high-order ghost imaging without a beam-splitter. The resolution of the PDCI is found to be $\frac{T}{N}$. So the imaging resolution can reach a micron or even smaller magnitude by setting experimental setup. The experimental operation in the PDCI is easier than that in the conventional ghost imaging. In the PDCI, the beam-splitter is not applied and the correlated detecting areas are arranged regularly, hence the correlated detecting areas can be matched easily. Moreover, a distinctive correlation microscopy can be realized by the PDCI, which is being investigated. Besides, the PDCI has a shape merging function, which adds some potential applications for the PDCI, such as the image reconstruction or the image encryption.

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